m-Best *S*-D Assignment Algorithm with Application to Multitarget Tracking

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In this paper we describe a novel data association algorithm, termed m-best S-D, that determines in $O(mSkn^3)$ time (m assignments, $S \geq 3$ lists of size n, k relaxations) the (approximately) m-best solutions to an S-D assignment problem. The m-best S-D algorithm is applicable to tracking problems where either the sensors are synchronized or the sensors and/or the targets are very slow moving. The significance of this work is that the m-best S-D assignment algorithm (in a sliding window mode) can provide for an efficient implementation of a suboptimal multiple hypothesis tracking (MHT) algorithm by obviating the need for a brute force enumeration of an exponential number of joint hypotheses.

We first describe the general problem for which the m-best S-D applies. Specifically, given line of sight (LOS) (i.e., incomplete position) measurements from S sensors, sets of complete position measurements are extracted, namely, the $1\text{st}, 2\text{nd}, \ldots, m$th best (in terms of likelihood) sets of composite measurements are determined by solving a static S-D assignment problem. Utilizing the joint likelihood functions used to determine the m-best S-D assignment solutions, the composite measurements are then quantified with a probability of being correct using a JPDA-like (joint probabilistic data association) technique. Lists of composite measurements from successive scans, along with their corresponding probabilities, are used in turn with a state estimator in a dynamic 2-D assignment algorithm to estimate the states of moving targets over time. The dynamic assignment cost coefficients are based on a likelihood function that incorporates the "true" composite measurement probabilities obtained from the (static) m-best S-D assignment solutions. We demonstrate the merits of the m-best S-D algorithm by applying it to a simulated multitarget passive sensor track formation and maintenance problem, consisting of multiple time samples of LOS measurements originating from multiple ($S = 7$) synchronized high frequency direction finding sensors.

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I. INTRODUCTION

A. Motivation

The problem of data association, namely, partitioning measurements across lists (e.g., sensor scans) into tracks and false alarms so that accurate estimates of true tracks can be recovered, has been extensively studied for many years. A probabilistic interpretation is typically given to the data association problem. Consequently, the assignment of measurements to tracks and false alarms is often done in a variety of ways. For multitarget tracking problems, a literature survey shows numerous well-known approaches proposed over the years, e.g., (in order of decreasing complexity) multiple hypothesis tracking (MHT) [5, 22, 43], multidimensional S-D ($S \geq 3$)[1] assignment [15, 16, 29, 30, 34, 35, 36, 37, 31], joint probabilistic data association (JPDA) [5], and two-dimensional (2-D) assignment (single scan processing) algorithms [1, 3, 6, 17, 20, 38–40].

Data association becomes especially difficult if the sensors are passive and measure line of sight (LOS) angles only for the targets. Measurements from multiple scans ($S \geq 3$) have to be associated to determine the estimates of target states, leading to a combinatorial explosion of the problem. Historically, the MHT algorithm has been considered to be the only approach that can truly provide an optimal data association solution. However, its practicality and feasibility have been hampered since it requires an enumeration of an exponentially increasing number of feasible joint association hypotheses to evaluate probabilities. Typically this is done within a time window and on a pruned set to limit its otherwise exploding computational requirements.

Data association using a multidimensional assignment algorithm such as S-D assignment [15, 16, 29, 30, 34–37] has been shown to be a practical and feasible alternative to MHT. S-D assignment is a discrete mathematical optimization formulation of the data association problem that systematically resembles an MHT within a window of length $(S-1)$. However, the main challenge to overcome in the S-D assignment problem is that of solving the ensuing NP-hard multidimensional assignment problem [31, 32]. In particular, an algorithm that determines the *optimal* solution is not only arduous, but also impractical for even fairly small sized problems, e.g., unsatisfactory results were reported for a problem as small as 10 targets and 3 scans in a dense scenario [30]. However, satisfactory tracking and computational performance

[1]For simplicity, unless otherwise stated, assume ($S \geq 3$) in all references to S-D.

can be realized utilizing existing S-D assignment algorithms that provide good *suboptimal* solutions, of quantifiable accuracy, and in pseudo-polynomial time [15, 16, 29, 30, 34–37].

In recent years, there has been considerable interest in an efficient and robust approach to data association based on an m-best assignment formulation [9–12, 28, 44]. With an appropriate modification of a cost matrix and by solving a series of modified copies of the initial problem, an algorithm, first due to Murty [27], can be used to find the m-best (ranked) solutions to not only the data association problem, but, in general, to many other classical optimization problems as well. However, in the context of the S-D assignment problem, determining the m-best solutions is more accurately stated as one of finding the m-best "approximate" solutions; finding the optimal solution to the S-D assignment problem is not feasible because it is NP-hard, consequently, finding suboptimal solutions, of quantifiable accuracy and in pseudo-polynomial time, is the only feasible alternative. Determining m-best solutions (as opposed to only the best one) becomes especially important for assignment-based approaches to data association since the *hard* irrevocable decisions that such approaches make can be mitigated via the m-best assignment formalism.

B. Related Research

The S-D assignment formulation of the data association problem is NP-hard for $S \geq 3$ even under the assumption of unity detection probability and no spurious measurements [19]. Thus, optimal solution techniques, based for example on branch and bound strategies for the set packing problem [26, 32], requiring unacceptably long times for 10 or more targets, are of little practical value [30]. There is a wide range of algorithms that can be used to construct suboptimal solutions to NP-hard combinatorial optimization problems. These include greedy heuristics, tabu search, simulated annealing, genetic algorithms, neural networks, and Lagrangian relaxation. The heuristic techniques with large data association errors are of little or no utility. For example, we showed in [16] that the greedy row-column heuristics can lead to data association error as high as 74%. Balas and Saltzman [2] developed greedy algorithms (diagonal, greedy, reduced cost, and max regret) along with a local search based on variable depth exchange. They conclude that the max regret method with variable depth exchange is a fast and quality suboptimal solution. In the context of multidimensional data association, Poore and Robertson [37] show that the max regret algorithms can, on the average, be off by as much as 15–68% from the optimal solutions. A customized greedy algorithm based on greedy randomized adaptive search procedure, termed GRASP [18, 23], had similar performance. Based on our preliminary (unpublished yet) studies, methods based on simulated annealing, genetic algorithms and neural networks are unlikely to be useful in real-time tracking situations.

Lagrangian relaxation-based methods have been found to perform well in tracking applications [15, 16, 21, 29–31, 34–37]. An advantage of this class of methods is that they provide both lower and upper bounds on the optimal solution and the difference between these bounds, termed the approximate duality gap, provides a measure of solution quality. Typical approximate duality gaps are in the 1–2% range. In the following, we present a relaxation algorithm that solves the S-D assignment problem as a sequence of relaxed subproblems.

Although the S-D assignment problem [1, 32] is NP-hard for $S \geq 3$, there exist techniques to solve the 2-D assignment algorithm in $O(n^3)$ time as discussed earlier. In [30], a 3-D assignment problem was solved as a series of 2-D subproblems, by relaxing the third constraint and appending it to the cost function using Lagrange multipliers. This resulted in a 2-D relaxed subproblem, which is easier to solve. The (relaxed) constraint is then reimposed to obtain a feasible solution that satisfies all the constraints. If this solution is suboptimal, the Lagrange multipliers (dual variables) are updated to penalize constraint violations, and the above process repeated. Thus, a hard constraint is transformed into a cost penalty.

However, when $S \geq 4$, multiple sets of constraints have to be relaxed. This can be accomplished in several ways.

In the method used by Poore, et al., [34] the constraints are relaxed one set at a time. Thus, the S-D problem is solved via Lagrangian relaxation by relaxing a set of constraints associated with the Sth list, solving the resulting $S-1$ dimensional subproblem iteratively and then reconstructing a feasible solution to the original S-D problem. The key requirement of the relaxation method is that the subproblem be optimally solvable, so that the approximate duality gap is guaranteed to be positive. This ensures that the reduced dimensional problem is a lower bound to the original problem. However, the $(S-1)$ dimensional subproblem is *not optimally solvable in polynomial time* if $S \geq 4$ (i.e., $S-1 \geq 3$). Poore's approach makes extensive use of partitioning techniques to break up the graph into smaller disjoint subgraphs, so that a Branch and Bound algorithm (which has *nonpolynomial* complexity) can be used to solve the $(S-1)$ dimensional subproblem optimally [34–36]. The approach has been successful for sparse graphs with minimal contention for measurements. However, it will take unacceptable computational resources for graphs that cannot be decomposed easily, such as the example considered here. For denser graphs, Poore suggests the use of a "merit function" [35] to address this difficulty. More recently, Poore [37] proposed a technique, which is similar in spirit to [13, 14, 16], of relaxing an S-D assignment problem to a 2-D one, optimizing with respect to the multipliers, and then recovering the feasible solution as an $S-1$ dimensional problem. This procedure is repeated until one reaches a 2-D recovery problem, which is solved optimally in polynomial time.

We adopt the approach developed by Pattipati and Deb [15, 16, 29, 30] of relaxing all the $(S-2)$D constraints simultaneously. The relaxed subproblem is then a 2-D assignment problem that can be optimally solved in pseudo-polynomial time. The challenge in this approach is to update the Lagrangian multipliers associated with the multiple constraint sets simultaneously for faster convergence. In [16], a novel and pseudo-polynomial time algorithm (irrespective of the sparsity of the graph) is presented where each constraint set is updated individually (via successive relaxation and constraint enforcement). The S-D assignment algorithm (similar in spirit to $S-1$ scan approximation widely used in MHT algorithm) extends naturally to tracking [5, 15, 16, 34, 35].

For research pertaining to m-best algorithms, Murty [27] was the first to recognize the utility of calculating not only the best (optimal) solution, but also the 2nd, 3rd, and, in general, the mth best solution to the 2-D assignment problem and various other classical optimization problems. Miller, et al. [25] proposed several optimizations to Murty's m-best 2-D assignment algorithm that substantially reduced its complexity from $O(mn^4)$ to $O(mn^3)$. For applications to multitarget tracking, Cox, et al. [10, 11] and Danchick, et al. [12] each proposed an m-best 2-D assignment algorithm. Nagarajan, et al. [28] and Brogan [9] separately presented similar branch-and-bound algorithms to determine the m-best 2-D assignments; however, their approaches were computationally inefficient and made no guarantee that the m-best assignments would indeed be determined. In [41], we developed several improvements to Cox's and Danchick's version of m-best 2-D, including: 1) a nonintrusive dynamic switching scheme between two different 2-D assignment algorithms, each highly suited for sparse and dense problems, respectively, and 2) a multilevel parallelization of the data association interface and m-best partitioning processes, respectively.

For data association problems, the manner in which the m-best solutions are processed allows for several data association approaches to be approximated [9–12, 28, 41, 42, 44]. Recent research has suggested that efficient MHT and JPDA solutions can be obtained when using an m-best 2-D assignment formulation of the data association problem [10–12]. However, because it lacks the time depth in lists (sensor scans) processed, i.e., 1 list processed at a time, an m-best 2-D algorithm is *only* a special case (1-scan) approximation of an MHT. However, a (1-scan) JPDA can be approximated using an m-best 2-D assignment algorithm. Alternatively, an m-best S-D, as proposed in this work, processes over S lists and is, in the authors' view, the correct way to approximate an $(S-1)$-scan MHT in the assignment framework. Moreover, an m-best 2-D is subsumed by an m-best S-D assignment algorithm.

C. Scope and Organization of Paper

The focus of this work is to describe the m-best S-D assignment algorithm and apply it to a track formation and maintenance passive sensor multitarget tracking problem. It should be noted that the m-best S-D assignment algorithm developed in this work is applicable to tracking problems where either the sensors are synchronized or the sensors and/or the targets are very slow moving.

Summarizing the algorithm, initially the m-best solutions to each of the different *static* S-D assignment problems are determined (in parallel) based on multiple time samples of measurements (direction of arrivals or LOS angles, i.e., incomplete position measurements) from S sensors. With imperfect detection probability (≤ 0.9) and low measurement accuracy ($\approx 2^\circ$ standard deviation), measurements from S sensors are to be associated to obtain composite measurements (i.e., complete target positions). For each of the derived composite measurements, JPDA-like probabilities that they represent true measurements are calculated. A series of *dynamic* 2-D assignment problems is then solved to track the motions of the targets across time using the composite measurement lists. In the 2-D assignment problem formulation, we assume nearly constant velocity motion for the targets, and subsequently use a Kalman filter (KF) for the state estimation problem. The likelihood function used to determine cost coefficients for the dynamic 2-D assignments utilizes the "true composite measurement probabilities" obtained from the (static) m-best S-D assignment solution.

In Section II we formulate the problem as just discussed. In Section III we describe the proposed m-best S-D assignment algorithm solution to the problem formulation. Section IV describes an efficient parallelization of the m-best S-D assignment algorithm, while Section V describes results of m-best S-D on a 7 sensor LOS problem. Section VI provides concluding remarks.

II. PROBLEM FORMULATION

In this section, we provide discussions for the following three subproblems: 1) the S-D assignment problem (a prelude to the next), 2) the m-best S-D assignment problem (used for track formation and generation of the composite measurements for track maintenance), and 3) the dynamic 2-D assignment problem for track maintenance.

A. S-D Assignment Problem

For the (static) S-D assignment problem[2] considered in this work, we are given S scans (lists) of measurements from N_S sensors monitoring a surveillance region, each with a number of detections, and not necessarily equal to the number of actual targets. In the present problem, $S = N_S$, i.e., there is one scan from each sensor, with all the time stamps identical within a scan (t) and the sensors are assumed synchronized (a reasonable assumption for the direction finders problem). The objective is to detect and localize an *unknown* number of targets by estimating their positions only using the S lists of measurements. The sensors in the surveillance region provide scans of detections at discrete time samples $t = 1,\ldots,T$. With each detection, there is an associated measurement, e.g., azimuth angle, or azimuth and elevation angles, depending on whether the sensor provides azimuth only or azimuth and elevation angles, respectively.

For each S-D assignment problem, we wish to associate the observations from S lists of n_s measurements obtained at time instant $t, s = 1,2,\ldots S$. Let the position of sensor s at t be $\mathbf{y}_s$ (for simplicity, the sensors are assumed fixed; otherwise, $\mathbf{y}_s = \mathbf{y}_s(t)$). The unknown position of target p is $\mathbf{x}_p$ at time t (omitted here for simplicity). The measurement $\mathbf{z}_{si_s}$, $i_s = 1,2,\ldots n_s$, (at time t) either originated from a true target p, in which case it is $H(\mathbf{x}_p,\mathbf{y}_{si_s})$ plus some additive white Gaussian noise $\mathcal{N}(0,\Sigma_s)$, or from some spurious source, in which case it is uniformly distributed within the field of view of sensor s. In addition, each sensor s has a known nonunity detection probability P_{D_s}.

Our goal is to identify (localize) the targets by providing estimates of their positions at time t. A generalized likelihood ratio which involves the target state estimates for the candidate associations is used to assign costs to each feasible S-tuple of measurements (candidate association) [5], and then an S-D assignment algorithm is used to *globally minimize* the cost. As mentioned before, a target may not be detected at every scan. To simplify the notation for incomplete measurement-to-target associations caused by missed detections, we add *dummy* measurements $\mathbf{z}_{s0}$ to each list. A dummy measurement from list s assigned to target p implies that this target was not detected by sensor s. The likelihood that an S-tuple of measurements $Z_{i_1 i_2 \ldots i_S}$, originated from target p, with the known state $\mathbf{x}_p$ at some instant t, is

$$\Lambda(Z_{i_1 i_2 \ldots i_S} \mid p) = \prod_{s=1}^{S} \{[1 - P_{D_s}]^{1-u(i_s)}[P_{D_s} p(\mathbf{z}_{si_s} \mid \mathbf{x}_p)]^{u(i_s)}\} \tag{1}$$

where $u(i_s)$ is an indicator function, i.e.,

$$u(i_s) = \begin{cases} 0 & \text{if} \quad i_s = 0 \\ 1 & \text{otherwise} \end{cases}. \tag{2}$$

The likelihood that the measurements are all spurious or *unrelated* to this target, i.e., $p = \emptyset$, is

$$\Lambda(Z_{i_1 i_2 \ldots i_S} \mid p = \emptyset) = \prod_{s=1}^{S} \left[\frac{1}{\Psi_s}\right]^{u(i_s)} \tag{3}$$

where Ψ_s is the volume[3] of the field of view of sensor s. The cost of associating the S-tuple to target p is given by the negative log-likelihood ratio[4]

$$c_{i_1 i_2 \ldots i_S} = -\ln \frac{\Lambda(Z_{i_1 i_2 \ldots i_S} \mid p)}{\Lambda(Z_{i_1 i_2 \ldots i_S} \mid p = \emptyset)}. \tag{4}$$

However, $\mathbf{x}_p$ in (1) is unknown, and, hence, will be replaced by its maximum likelihood (ML) estimate, i.e.,

$$\hat{\mathbf{x}}_p = \arg\max_{\mathbf{x}_p} \Lambda(Z_{i_1 i_2 \ldots i_S} \mid p) \tag{5}$$

which makes (4) into a generalized likelihood ratio. Substituting (1) and (5) in (4), the *cost* of the candidate association of the S-tuple of measurements $(i_1, i_2, \ldots, i_S)$ to a target is

$$\begin{aligned} c_{i_1 i_2 \ldots i_S} = \sum_{s=1}^{S} \Bigg\{ & [u(i_s) - 1]\ln(1 - P_{D_s}) \\ & - u(i_s) \ln\left(\frac{P_{D_s}\Psi_s}{|2\pi\Sigma_s|^{1/2}}\right) + u(i_s) \\ & \times (\tfrac{1}{2}[\mathbf{z}_{si_s} - H(\hat{\mathbf{x}}_p,\mathbf{y}_{si_s})]^T \Sigma_s^{-1} \\ & \times [\mathbf{z}_{si_s} - H(\hat{\mathbf{x}}_p,\mathbf{y}_{si_s})]) \Bigg\}. \end{aligned} \tag{6}$$

Note we do not use a sensor-specific false alarm rate in the negative likelihood expression. Our goal is to find the most likely set of S-tuples such that each measurement is assigned to one and only one target, or declared false, and each target receives

[2]The more accurate notation would be the S-D (t) assignment problem since we determine the m-best solutions to $t = 1,\ldots,T$ static S-D assignment problems. However, we use the simpler notation to avoid unnecessary clutter.

[3]If the false alarm probability is zero, Ψ_s is the volume of the field of view of sensor s or some scaled version of it. Using a uniform pdf amounts to a scaling of the likelihood ratio and, hence, its exact value is not relevant to the maximization.

[4]Since the likelihood function (LF), being a pdf, has a physical dimension, one cannot compare for example the LF of 2 measurements with the LF of 3 measurements. Such a comparison is possible only by using likelihood ratios, since they are dimensionless quantities [5].

at most one measurement from each list. This can be reformulated as the following (generalized) S-D *assignment problem*, i.e.,

$$\min_{\rho_{i_1 i_2 \ldots i_S}} \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \cdots \sum_{i_S=0}^{n_S} c_{i_1 i_2 \ldots i_S} \rho_{i_1 i_2 \ldots i_S} \tag{7}$$

subject to:

$$\sum_{i_2=0}^{n_2} \cdots \sum_{i_S=0}^{n_S} \rho_{i_1 i_2 \ldots i_S} = 1, \qquad i_1 = 1,2,\ldots,n_1$$

$$\sum_{i_1=0}^{n_1} \sum_{i_3=0}^{n_3} \cdots \sum_{i_S=0}^{n_S} \rho_{i_1 i_2 \ldots i_S} = 1, \qquad i_2 = 1,2,\ldots,n_2$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$\sum_{i_1=0}^{n_1} \cdots \sum_{i_{S-1}=0}^{n_{S-1}} \rho_{i_1 i_2 \ldots i_S} = 1, \qquad i_S = 1,2,\ldots,n_S$$

where, $\{\rho_{i_1 i_2 \ldots i_S}\}$ are binary association variables such that $\rho_{i_1 i_2 \ldots i_S} = 1$ if the S-tuple $Z_{i_1 i_2 \ldots i_S}$ is associated with a candidate target. Otherwise, it is set to zero.

Note that there are no constraints placed on the dummy measurements. Hence, this is a *generalized* S-D assignment problem. In addition, through the use of the dummy measurements, the association is now performed over sets of complete S-tuples. Thus, if a target was missed in list s, the corresponding S-tuple has $i_s = 0$. However, a candidate target requires a minimum number of true (nondummy) measurements. This is discussed in more detail in Section V.

B. m-best S-D Assignment Problem

For the m-best S-D assignment problem considered in this work, we wish to determine the m-best solutions to a given static S-D assignment problem as just described. To determine the m-best assignments, we determine and rank (in polynomial time) the S-D assignment problem solutions in order of increasing cost. The manner in which this is done is by utilizing a generalized form of Murty's m-best 2-D assignment *partitioning* process [27].

Recall that each static S-D assignment problem consisted of S lists with n_s measurements in list $s = 1,\ldots,S$. For the m-best S-D assignment problem, let us denote each static S-D assignment problem,[5] say $\mathcal{D}$, by its list of S-tuples of measurements and their corresponding association costs, i.e.,

$$\begin{aligned} &\mathcal{D} = \{\langle (i_1,\ldots,i_S), c_{i_1\ldots i_S}\rangle\} \\ &i_s = 1,2,\ldots n_s, \qquad s = 1,2,\ldots S. \end{aligned} \tag{8}$$

Each S-tuple $(i_1,\ldots,i_S)$ represents a hypothesized localized target, having position estimate $\hat{\mathbf{x}}_p$ given by (5), and a corresponding negative log-likelihood cost $c_{i_1\ldots i_S}$ given by (4). A feasible solution, or assignment, say a_i, is a set of S-tuples in which each i_s appears exactly once (except for dummy measurements 0_s which may appear multiple times), i.e.,

$$\begin{aligned} a_i &= \bigcup_{j=1}^{M} \{(i_{1_j},\ldots,i_{S_j})\} \\ M &= \prod_{s=1}^{S} (n_s + 1) \\ i_{s_k} &\neq i_{s_l}, \qquad k \neq l, \quad i_s = 1,\ldots,n_s. \end{aligned} \tag{9}$$

The feasible solution space A can then be expressed as

$$A = \bigcup \{a_i\}. \tag{10}$$

The cost of an assignment (or hypothesis), denoted by $c(a_i)$, can then be determined by summing the individual costs (negative log-likelihoods) corresponding to the S-tuples occurring in the assignment, i.e.,

$$c(a_i) = \sum_{(i_1,\ldots,i_S)\in a_i} c_{i_1\ldots i_S} \tag{11}$$

Determining the single best (most likely) assignment $a^*_{(1)}$ to $\mathcal{D}$ is then a matter of determining the assignment that minimizes this sum. In this work, the most likely assignment is determined by solving (suboptimally) the static S-D assignment problem using the S-D assignment algorithm developed in [16] and outlined in the previous subsection.

The m-best assignments to $\mathcal{D}$, i.e., $a^*_{(1)},\ldots,a^*_{(m)}$, are the m assignments $a_i \in A$ with the m least costs, i.e.,

$$a^*_{(1)} = \arg\min_{a_i\in A} \{c(a_i)\} \tag{12}$$

$$a^*_{(2)} = \arg\min_{a_i\in A\backslash a^*_{(1)}} \{c(a_i)\} \tag{13}$$

$$\vdots$$

$$a^*_{(m)} = \arg\min_{\substack{a_i\in A\backslash a^*_{(k)} \\ k=1,\ldots,m-1}} \{c(a_i)\}. \tag{14}$$

C. Dynamic 2-D Assignment Problem

In the dynamic 2-D assignment problem, we want to track an unknown number of targets by estimating their states (positions and velocities) using the t composite measurement lists determined by way of solving the t (static) m-best S-D assignment problems. Actually, in a real-time situation, the dynamic problem is solved after each scan to update

[5] Similarly as for the S-D assignment problem, the more accurate notation would be $\mathcal{D}(t)$, $t = 1,\ldots,T$, since we determine the m-best solutions to T such S-D assignment problems. However, we again use the simpler notation to avoid unnecessary clutter.

the tracks, starting with the second scan (i.e., the second m-best S-D problem). We assume the target state evolves according to a known linear dynamic model, corrupted by process noise, i.e.,

$$\mathbf{x}(t) = F(\delta)\mathbf{x}(t-1) + G(\delta)v(t-1) \tag{15}$$

where, in the present work, the target (or track) state is a second-order kinematic model with two- or three-dimensional coordinates, δ is the time interval between scans, $F(\cdot)$ is the state transition matrix, $G(\cdot)$ is the disturbance matrix, and $v(\cdot)$ is zero-mean, white process noise with (known) covariance matrix $Q(\cdot)$. The composite measurements (if true) are assumed to be linear functions of the target state corrupted by measurement noise, i.e.,

$$\mathbf{z}(t) = H\mathbf{x}(t) + w(t) \tag{16}$$

where H is the measurement matrix, and $w(\cdot)$ is zero-mean, white measurement noise with (known) covariance matrix $R(\cdot)$. This latter covariance follows from the static association of LOS sensor measurements and is presented in Appendix A.

Before describing the dynamic 2-D assignment problem formulation, we first describe the derivation of the composite measurements and their corresponding probabilities of being true. We use an efficient (nonenumerative) extraction technique for the composite measurements and determine their probabilities using the likelihoods associated with the feasible *joint events* (assignments) corresponding to the m-best S-D assignment solutions.

Denote the list of composite measurements (S-tuples) corresponding to the tth (static) m-best S-D assignment problem solution by

$$\Phi(t) = \bigcup_{k=1}^{m} a^*_{(k)} \tag{17}$$

where

$$|a^*_{(1)}| \leq |\Phi(t)| \leq \sum_{k=1}^{m} |a^*_{(k)}|. \tag{18}$$

Note $|\Phi(t)|$ is the cardinality of set $\Phi(t)$, i.e., the number of distinct S-tuples (composite measurements) in $\Phi(t)$. For each composite measurement $z \in \Phi(t)$, define the binary indicator variable

$$d_{zk} = \begin{cases} 1 & \text{if} \quad z \in a^*_{(k)} \\ 0 & \text{otherwise} \end{cases}. \tag{19}$$

To quantify a composite measurement's *true measurement pseudoprobability* (the approximate probability of being correct), we use a JPDA-like technique [5], i.e.,

$$P_z = \frac{\sum_{k=1}^{m} e^{-[c(a^*_{(k)}) - c(a^*_{(1)})]} d_{zk}}{\sum_{k=1}^{m} e^{-[c(a^*_{(k)}) - c(a^*_{(1)})]}} \tag{20}$$

where $c(a_{(k)})$ is defined by (11) and denotes the cost of the kth best assignment (or hypothesis), i.e., the sum of the individual costs (negative log-likelihoods) that correspond to the S-tuples occurring in the kth best assignment. Note that the best assignment cost $c(a^*_{(1)})$ is used as a normalization in the composite measurement probability quantification since: 1) it is necessary to avoid the numerical errors that would ensue when raising the exponential e to such large powers, and 2) it provides a mechanism to threshold the (dynamic) number of m-best assignments determined per S-D assignment problem, i.e, we determine the m-best solutions to an S-D assignment problem satisfying the following difference constraint,

$$[c(a^*_{(k)}) - c(a^*_{(1)})] < \delta \tag{21}$$

where the threshold δ is an input parameter and is such that $e^{-\delta} \approx \epsilon$, where ϵ is set equal to the computer's precision.

Once the composite measurement lists have been compiled and their respective probabilities determined, we are then ready to enter the dynamic 2-D assignment problem phase for track maintenance. We cast the 2-D assignment problem as follows: $|\Phi(t)|$ composite measurements from the latest scan list t are to be assigned to the $N(t-1)$ most likely tracks from the previous scans using a global cost minimization function [4, 5] based on likelihood ratios and composite measurement probabilities. Specifically, let y_i, $i = 0,\ldots,N(t-1)$ denote a particular track from the set of existing tracks (including a *dummy* track y_0), and z_j, $j = 0,\ldots,|\Phi(t)|$ denote a particular measurement from the latest scan list t of composite measurements (including a *dummy* composite measurement z_0). Define the binary assignment variable

$$\chi_{y_i z_j} = \begin{cases} 1 & \text{if} \quad z_j \text{ is assigned to } y_i \\ 0 & \text{otherwise} \end{cases}. \tag{22}$$

Note that $\chi_{y_i z_0} = 1$ implies that track y_i is unassociated and has missed a detection in the latest scan. Furthermore, $\chi_{y_0 z_j} = 1$ implies that composite measurement z_j is unassociated, i.e., not assigned to any of the $N(t-1)$ existing tracks, but, instead, assigned to the dummy track (false alarm or new track initiation). Since measurement errors within a scan are independent of each other, maximizing the likelihood ratio, consisting of the joint pdf-probability [4, 5] of measurements given their origins and the corresponding detection events, over the set of feasible assignments can be cast into the following 2-D assignment problem:

$$\min \sum_{y_i=0}^{N(t-1)} \sum_{z_j=0}^{|\Phi(t)|} c_{y_i z_j} \chi_{y_i z_j} \tag{23}$$

subject to:

$$\sum_{z_j=0}^{|\Phi(t)|} \chi_{y_i z_j} = 1, \qquad y_i = 1,\ldots,N(t-1)$$

$$\sum_{y_i=0}^{N(t-1)} \chi_{y_i z_j} = 1, \qquad z_j = 1,\ldots,|\Phi(t)|$$

where the cost of assigning measurement z_j to track y_i is

$$c_{y_i z_j} = \begin{cases} 0 & \text{if} \quad y_i \text{ or } z_j = 0 \\ -\log\left(\dfrac{\Lambda(y_i,z_j)P_{z_j}}{\Lambda(0,z_j)}\right) & \text{if} \quad -\log(\cdot) < 0 \\ \infty & \text{otherwise} \end{cases} \cdot \tag{24}$$

The numerator in the above $-\log(\cdot)$ expression is based on a likelihood function (LF) calculation $\Lambda(\cdot)$ from a KF state estimator and the composite measurement probability given by (20). The numerator denotes the likelihood that composite measurement z_j from the tth (static) m-best S-D assignment problem solution originated from track y_i, and the denominator is the likelihood that measurement z_j corresponds to none of the existing tracks (i.e., a false alarm). The likelihood of false alarms, i.e., $\Lambda(0,z_j)$, is assumed uniformly probable over each sensor's field of view [5].

III. ALGORITHM DESCRIPTION

For the three subproblems identified in the previous section, below we describe our solution approaches for each, namely, 1) the static S-D assignment solution, 2) the m-best S-D assignment solution, and 3) the dynamic 2-D assignment solution.

To summarize the algorithm, initially sets of measurements from each sensor are associated (accounting for missed detections and false alarms) using a static S-D assignment algorithm based on [15, 16, 29, 30]. Afterwards, the m-best S-D assignment solutions are determined and combined (using an approach based on [41, 42]) to form composite measurements based on a JPDA approach. Finally, these composite measurements are passed to a dynamic 2-D assignment algorithm using an approach based on [6, 30].

A. Static S-D Assignment Solution

Using the approach described in [15, 16, 29, 30], the S-D assignment problem is solved as a series of relaxed 2-D subproblems in two phases: 1) the constraint relaxation phase, and 2) the multiplier update and constraint enforcement phase. We successively relax the constraint sets $r = S, S-1, S-2,\ldots,3$ and append them to the cost function using *Lagrangian* multipliers $\mathbf{u}_r$. Thus, at stage $r = 3$ we have relaxed the problem to a 2-D assignment problem, which is then optimally solved using the generalized auction algorithm [6, 15]. In the constraint enforcement phase, we compute a feasible (but most likely suboptimal) solution to the 3-D problem via a 2-D assignment problem by enforcing the third constraint set, and update $\mathbf{u}_3$, via an accelerated subgradient method [33]. Similarly, we successively compute feasible solutions to the r-D subproblems for $r = 4,5,\ldots,S$ via a 2-D assignment problem by enforcing the rth constraint; and update the corresponding Lagrangian multiplier vectors $\mathbf{u}_r$. This cycle of relaxing constraints, solving reduced dimensional assignment problems and updating the Lagrangian multipliers is repeated until all the constraints are satisfied in the relaxed problem (in which case the solution is optimal), or the feasible solution is of acceptable quality. For more details on this approach, see [16].

B. m-best S-D Assignment Solution

To determine the m-best solutions to a static S-D assignment problem, we use an approach similar to the one we used in our m-best 2-D assignment work [41, 42]. Specifically, given an S-D assignment problem, denoted as $\mathcal{D}$, we partition the (sub)problem $\mathcal{D}_{(m-1)}$ that determines the $(m-1)$st best S-D assignment $a^*_{(m-1)}$ into $n = 1,\ldots,|a^*_{(m-1)}|$ subproblems $\mathcal{D}_{k_n}$ $(k = m-1)$ having solution subspaces $A_{k_n} \subset A$ and enforce the following two constraints:

$$\bigcup_{n=1}^{|a^*_{(k)}|} A_{k_n} = A_k - a^*_{(k)} \tag{25}$$

$$A_{k_i} \cap A_{k_j} = \emptyset \qquad i,j = 1,\ldots,|a^*_{(k)}|, \quad i \neq j. \tag{26}$$

To create subproblem $\mathcal{D}_{k_1}$, we first copy $\mathcal{D}_k$ to $\mathcal{D}_{k_1}$. We then remove from $\mathcal{D}_{k_1}$ the 1st S-tuple in the best (most likely) assignment $a^*_{(k)}$ of $\mathcal{D}_k$, i.e., we remove $(i_{1_1},\ldots,i_{S_1}) \in a^*_{(k)}$ from $\mathcal{D}_{k_1}$. Hence, subproblem $\mathcal{D}_{k_1}$ is $\mathcal{D}_k$ less $(i_{1_1},\ldots,i_{S_1})$, which implies that no solution to $\mathcal{D}_{k_1}$ will ever contain this 1st S-tuple in its solution space, i.e.,

$$A_{k_1} = \{a_i \in A_k : (i_{1_1},\ldots,i_{S_1}) \notin a_i\} \tag{27}$$

In general, to create subproblem $\mathcal{D}_{k_n}$, $2 \leq n \leq |a^*_{(k)}|$, we first copy $\mathcal{D}_k$ to $\mathcal{D}_{k_n}$, and then perform the following two steps. First, we remove from $\mathcal{D}_{k_n}$ the nth S-tuple in the best assignment $a^*_{(k)}$ of $\mathcal{D}_k$, i.e., we remove $(i_{1_n},\ldots,i_{S_n}) \in a^*_{(k)}$ from $\mathcal{D}_{k_n}$. This implies that no solution to $\mathcal{D}_{k_n}$ will ever contain this nth S-tuple in its solution space. Second, we enforce the 1st,$\ldots,(n-1)$st S-tuples, i.e., $(i_{1_j},\ldots,i_{S_j}) \in a^*_{(k)}$, $j = 1,\ldots,(n-1)$, in the best assignment of $\mathcal{D}_k$ to be in all solutions to $\mathcal{D}_{k_n}$. We enforce this by removing in $\mathcal{D}_{k_n}$ all S-tuples incident to i_{s_j}, $s = 1,\ldots,S$ in $\mathcal{D}_k$, except for the S-tuples $(i_{1_j},\ldots,i_{S_j})$ themselves, which implies that every solution to $\mathcal{D}_{k_n}$ will contain the 1st,…,$(n-1)$st S-tuples in its solution space. Hence,

$$A_{k_n} = \left\{ \begin{array}{l} a_i \in A_k : (i_{1_j},\ldots,i_{S_j}) \in a_i, \\ (i_{1_n},\ldots,i_{S_n}) \notin a_i, \\ (i_{1_j},\ldots,i_S),\ldots,(i_1,\ldots,i_{S_j}) \notin a_i \end{array} \right\}. \tag{28}$$

Note that solution spaces A_{k_n} for subproblems $\mathcal{D}_{k_n}$, $n = 1,\ldots,|a^*_{(k)}|$, are disjoint and their union will be exactly the solution space to $\mathcal{D}_k$ less its best assignment (i.e., $A_k - a^*_{(k)}$).

After partitioning $\mathcal{D}_k$ according to its best assignment $a^*_{(k)}$, we solve each subproblem $\mathcal{D}_{k_n}$, $1 \le n \le |a^*_{(k)}|$, and pair it together with its best solution $a_{(k_n)}$, and place each pairing $(\mathcal{D}_{k_n}, a_{(k_n)})$ on a priority queue, say Q. The mth best assignment for $\mathcal{D}$, i.e., $a^*_{(m)}$, is the assignment $a_{(j_n)}$ $(j = 1,\ldots,m-1)$ corresponding to subproblem $\mathcal{D}_{j_n}$ on the queue Q having minimum cost (or maximum likelihood), i.e.,

$$a^*_{(m)} = \arg \min_{a_{(j_n)} \in Q} \{c(a_{(j_n)})\}. \tag{29}$$

The complexity of our m-best S-D assignment algorithm is as follows. Per S-D assignment problem, we perform one partitioning task for each of the m-best assignments determined. Hence, in the worst case, each partitioning creates n new subproblems, where $n = |a^*_{(m-1)}|$. This creates up to $O(mn)$ S-D assignment problems to solve and insertions of (problem, solution) pairings in the queue Q. Solving each S-D assignment problem has worst case complexity $O(Skn^3)$ [16], where k is the number of relaxation iterations. Each insertion step has worst case complexity $O(mn)$. Hence, the worst case complexity of m-best S-D is $O(mn(Skn^3 + mn)) = O(mSkn^4)$. Using the preprocessing and optimization steps as proposed by Miller [25] and in a previous effort of ours [41], the complexity of the m-best S-D assignment algorithm can be reduced to $O(mSkn^3)$.

C. Dynamic 2-D Assignment Solution

In this work, we used the generalized auction algorithm [6, 30] to solve the $t = 1,\ldots,T$ dynamic 2-D assignment problems. Since the auction algorithm is well known, we do not describe it here and simply refer interested readers to [6, 30] for more details.

IV. ALGORITHM PARALLELIZATION

To help mitigate the computational complexity issues associated with the m-best S-D assignment algorithm, we developed a multiprocess parallelization on a 4-processor shared-memory SPARCstation 20 multiprocessor. The software utilized in this work consisted of the Solaris 2.4.2 development environment and the SunOS 5.4.2 UNIX operating system (OS). For this parallelization, we utilized UNIX's shared-memory interprocess communication (IPC) constructs for our synchronization mechanism.

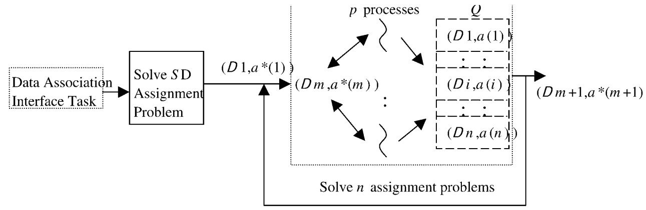

Fig. 1. Shared-memory parallelization of partitioning task in m-best S-D.

The parallelization developed for m-best S-D exploits its many independent and highly parallelizable tasks, i.e., all tasks associated with solving the multiple 2-D assignment problems generated as a result of the m-best partitioning process. The parallelization is coarse-grained and based on the supervisor/worker model (see Fig. 1). Recall that in the partitioning task, after determining the best (most likely) solution to an initial S-D assignment problem, denoted as $(D, a^*_{(1)})$, the partitioning task consists of creating $n = |a^*_{(1)}|$ subproblems, say $D_1,\ldots,D_n$, and determining their best solutions, say $a_{(1)},\ldots,a_{(n)}$, respectively. Since each of the n subproblems are independent of one another, they can be processed (i.e., created and solved) in parallel.

Specifically, the supervisor process creates a specified number of worker processes, say $p \le n$, to process the n subproblems and waits for the processing to be completed before determining the next best assignment. Each worker process, asynchronously and in parallel, creates its respective subproblem(s) and determines the (their) best solution(s), i.e., collectively they determine $(D_1, a_{(1)}),\ldots,(D_n, a_{(n)})$. Since the processing cost corresponding to each subproblem is not uniform (depends on the number of tuples that are enforced and/or removed based on the partitioning process), dynamic scheduling of subproblems across processes is employed. In this way, maximum load balancing is achieved [38]. Upon processing of the n subproblems by the p worker processes, the supervisor process can then determine the 2nd best assignment and the corresponding subproblem that determines it, say, $(D_j, a_{(j)} = a^*_{(2)})$, $1 \le j \le n$. To find the 3rd best assignment, we simply repeat this process, replacing $(D, a^*_{(1)})$ with $(D_j, a^*_{(2)})$, and so on.

V. RESULTS

A. Problem Description

In this section, we solve at $t = 1,\ldots,10$ different time instances the following problem. There are N_S

Composite Measurements, Corresponding True Measurement Probabilities, True Origin at Time $t = 1$

Meas. No.	S-tuple $z_i \in \Phi(t)$	ML Pos Est $\hat{\mathbf{x}}_p$	Neg Log-Like c_{z_i}	Probability P_z	LOS Components Target ID–FA	Composite Meas. Origin
1	(1,1,0,1,1,1,1)	[−813.66,524.19]	−102.96	0.000003	t1,t1,0,t1,t1,t1,t1	—
2	(2,2,1,2,2,2,2)	[−367.91,545.97]	−115.46	0.000003	t2,t2,t2,t2,t2,t2,t2	—
3	(3,3,2,3,3,3,3)	[−5.93,507.58]	−108.57	1.0	t3,t3,t3,t3,t3,t3,t3	t3
4	(4,5,3,4,4,4,4)	[437.00,464.54]	−94.11	1.0	t5,t5,t4,t4,t4,t4,t4	t4,t5 {Mx}
5	(0,4,0,5,5,5,0)	[833.54,890.15]	−17.55	0.000003	0,t4,0,t5,t5,t5,0	—
6	(2,2,1,2,2,2,3)	[−349.44,587.77]	−114.26	0.0	t2,t2,t2,t2,t2,t2,t3	—
7	(3,3,2,3,3,3,2)	[−22.58,478.32]	−102.86	0.0	t3,t3,t3,t3,t3,t3,t2	—
8	(1,1,0,1,1,1,2)	[−802.62,580.31]	−95.48	0.999997	t1,t1,0,t1,t1,t1,t2	t1
9	(2,2,1,2,2,2,1)	[−378.46,513.66]	−115.29	0.999997	t2,t2,t2,t2,t2,t2,t1	t2

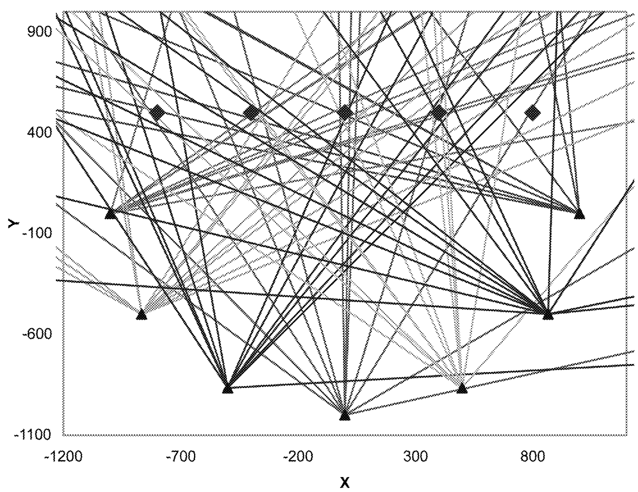

Fig. 2. Target positions and LOS measurements for 7 sensor 5 target direction finding problem at time sample $k = 1$.

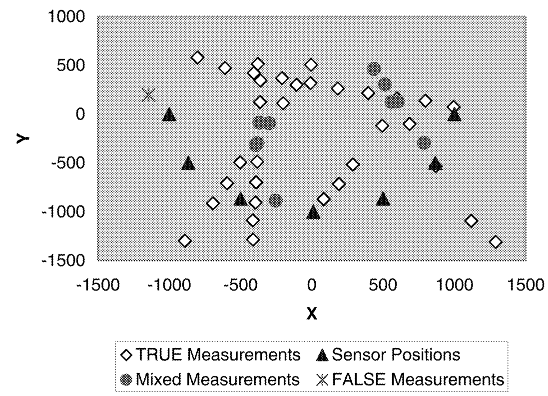

Fig. 3. Composite measurements for 7 sensor 5 target direction finding problem across $k = 1, \ldots 10$ time samples.

sensors at known fixed locations in a plane arranged in a semicircle of radius 1000 km centered at the origin of the coordinate system. Each sensor is a direction finding ionosonde [8], i.e., a multielement passive interferometer which estimates the LOS (azimuth) of an RF emission, for example, a radio message from an aircraft, via (DF) ionogram. These sensors have extremely high range and can "see" beyond the curvature of the Earth, because they receive the signals bouncing off the ionosphere. However, because of the time-varying instabilities in the ionosphere, the rms accuracy of these measurements varies widely (0.3° to 1.7° [8]) depending on the prevalent climatic conditions, time of the day and a variety of other factors. In our simulations we used LOS measurement error standard deviation, σ_θ, of 2.0°. The sensors are assumed to be forward looking with a field of view of 180°, with detection probability of $P_D = 0.9$. The false alarm rate of the sensors is 0.8/rad. With 5 targets, the average number of detections per scan is therefore 7 (with $5P_D = 4.5$ true detections and $0.8\pi = 2.5$ spurious detections).

This scenario results in a rather significant number of candidate associations to process. In Fig. 2 we illustrate the numerous LOS measurements generated for one specific time sample. Any two LOS measurements intersect at a point in a plane, implying a target at that position would produce these two measurements. To reduce the number of candidate associations, we require that a target must be detected by the majority of the sensors to be considered in the association process. Thus, in the 7 sensor scenario, a candidate association must include at least 4 nondummy LOS measurements. Note that a true target is detected by 4 or more sensors in a 7 sensor scenario with a probability of 0.997. If there are only three sensors, a true target is detected by 2 or more sensors with a probability of 0.97. Therefore, this assumption does not lead to a significant loss of accuracy.

The corresponding composite measurements determined as a result of solving for the 30-best static S-D ($S = 7$) assignment problems for each of the 10 different time samples as shown in Fig. 3. In Tables I–X we provide, for each composite measurement determined in each of the 10 time samples, the composite measurement's 1) S-tuple, 2) ML position estimate, 3) negative log-likelihood cost, 4) true measurement probability, 5) the actual target ID (or false alarm) that determined each LOS measurement in the corresponding S-tuple for the composite measurement, and 6) the origin of the

TABLE II
Composite Measurements, Corresponding True Measurement Probabilities, True Origin at Time $t = 2$

Meas. No.	S-tuple $z_i \in \Phi(t)$	ML Pos Est $\hat{\mathbf{x}}_p$	Neg Log-Like c_{z_i}	Probability P_z	LOS Components Target ID–FA	Composite Meas. Origin
1	(1,1,0,1,1,1,3)	[−610.82,468.43]	−102.52	0.0	t1,t1,0,t1,t1,t1,t3	—
2	(2,2,1,2,2,2,1)	[−360.22,350.67]	−119.53	0.0	t2,t2,t2,t2,t2,t2,t1	—
3	(3,3,2,3,3,3,2)	[−107.05,300.86]	−109.02	1.0	t3,t3,t3,t3,t3,t3,t2	t3
4	(4,5,3,4,4,4,4)	[514.02,305.47]	−107.89	1.0	t5,t5,t4,t4,t4,t4,t4	t4,t5 {Mx}
5	(0,4,0,5,5,5,0)	[689.95,455.64]	−21.67	0.0	0,t4,0,t5,t5,t5,0	—
6	(1,1,0,1,1,1,1)	[−609.58,472.43]	−103.17	1.0	t1,t1,0,t1,t1,t1,t1	t1
7	(2,2,1,2,2,2,3)	[−360.70,346.06]	−118.81	1.0	t2,t2,t2,t2,t2,t2,t3	t2

TABLE III
Composite Measurements, Corresponding True Measurement Probabilities, True Origin at Time $t = 3$

Meas. No.	S-tuple $z_i \in \Phi(t)$	ML Pos Est $\hat{\mathbf{x}}_p$	Neg Log-Like c_{z_i}	Probability P_z	LOS Components Target ID–FA	Composite Meas. Origin
1	(1,1,0,1,3,1,1)	[−401.17,426.47]	−105.12	0.000027	t1,t1,0,t1,t3,t1,t1	—
2	(2,2,1,2,2,2,3)	[−363.17,123.96]	−120.11	1.0	t2,t2,t2,t2,t2,t2,t3	t2
3	(3,3,2,3,1,3,2)	[−206.32,109.53]	−108.78	0.000027	t3,t3,t3,t3,t1,t3,t2	—
4	(4,5,3,4,4,5,4)	[605.73,128.19]	−120.75	1.0	t5,t5,t4,t4,t4,t5,t4	t4,t5 {Mx}
5	(0,4,0,5,5,4,0)	[560.36,124.10]	−24.58	1.0	0,t4,0,t5,t5,t4,0	t4,t5 {Mx}
6	(1,1,0,1,1,1,1)	[−407.60,421.75]	−103.32	0.999973	t1,t1,0,t1,t1,t1,t1	t1
7	(3,3,2,3,3,3,2)	[−200.33,112.72]	−109.58	0.999973	t3,t3,t3,t3,t3,t3,t2	t3

TABLE IV
Composite Measurements, Corresponding True Measurement Probabilities, True Origin at Time $t = 4$

Meas. No.	S-tuple $z_i \in \Phi(t)$	ML Pos Est $\hat{\mathbf{x}}_p$	Neg Log-Like c_{z_i}	Probability P_z	LOS Components Target ID–FA	Composite Meas. Origin
1	(1,1,0,1,1,1,1)	[−208.65,370.19]	−103.38	0.999999	t1,t1,0,t1,t1,t1,t1	t1
2	(2,2,1,2,2,3,3)	[−366.73,−86.12]	−116.59	1.0	t2,t2,t2,t2,t2,t3,t3	t2,t3 {Mx}
3	(3,3,2,3,3,2,2)	[−299.43,−91.48]	−114.34	1.0	t3,t3,t3,t3,t3,t2,t2	t2,t3 {Mx}
4	(4,5,3,5,5,5,4)	[493.78,−118.32]	−119.13	1.0	t5,t5,t4,t5,t5,t5,t4	t5
5	(0,4,0,4,4,4,0)	[685.75,−99.26]	−12.45	0.999999	0,t4,0,t4,t4,t4,0	t4
6	(2,2,1,2,2,3,2)	[−365.72,−81.29]	−113.26	0.0	t2,t2,t2,t2,t2,t3,t2	—
7	(3,3,2,3,3,2,3)	[−300.17,−91.16]	−116.60	0.0	t3,t3,t3,t3,t3,t2,t3	—
8	(4,5,3,5,5,5,0)	[493.61,−106.97]	−50.70	0.0	t5,t5,t4,t5,t5,t5,0	—
9	(0,4,0,4,4,4,4)	[680.22,−91.30]	−48.08	0.0	0,t4,0,t4,t4,t4,t4	—
10	(3,3,2,3,3,2,4)	[−299.63,−121.72]	−89.67	0.0	t3,t3,t3,t3,t3,t2,t4	—
11	(4,5,3,5,5,5,2)	[480.91,−55.44]	−104.49	0.0	t5,t5,t4,t5,t5,t5,t2	—
12	(1,1,0,1,1,1,0)	[−208.78,369.90]	−42.39	0.000001	t1,t1,0,t1,t1,t1,0	—

composite measurements which have nonnegligible probability of being true according to (20). While some composite measurements are "mixed" (the equivalent of unresolved targets), only one is a "false alarm" (see the {Mx}and {FA}references in Tables I–X for mixed and false alarm measurements, respectively).

For the simulation, the 5 targets were initially placed on the $y = 500$ line, with an intertarget separation of 400 km. For $\sigma_\theta = 2^\circ$, the intertarget separation, as seen by the middle sensor is 7.5 standard deviations. The targets are well separated so that there are no unresolved targets. Nevertheless, there are numerous candidate "ghost" intersections of LOS measurements as well as a large number of possible subsets of these intersections at each scan, which the 7D assignment problem tackles successfully with only a few incorrect composite measurements being accepted.

Fig. 4 shows the true track trajectories and corresponding estimated track state predictions for each of the 5 targets in track. Note that no false or missing tracks were generated by the m-best S-D tracking algorithm for this tracking scenario. Also, as is clearly seen from Fig. 4, fairly accurate data association and state estimation ensued during the dynamic 2-D phase that follows the m-best S-D. One major reason for this was because much of the work and cleaning up of the data was accomplished during the static m-best S-D phase, i.e., instead of trying to associate and filter S sets of LOS measurements from T different time samples, we associate and filter T

Composite Measurements, Corresponding True Measurement Probabilities, True Origin at Time $t = 5$

Meas. No.	S-tuple $z_i \in \Phi(t)$	ML Pos Est $\hat{\mathbf{x}}_p$	Neg Log-Like c_{z_i}	Probability P_z	LOS Components Target ID–FA	Composite Meas. Origin
1	$(1,1,0,1,1,1,1)$	$[-8.76, 318.96]$	-103.38	1.0	t1,t1,0,t1,t1,t1,t1	t1
2	$(2,2,2,2,2,3,3)$	$[-380.67, -298.34]$	-114.64	1.0	t2,t2,t3,t2,t2,t3,t3	t2,t3 {Mx}
3	$(3,3,1,3,3,2,2)$	$[-392.08, -314.85]$	-112.67	1.0	t3,t3,t2,t3,t3,t2,t2	t2,t3 {Mx}
4	$(4,5,3,4,4,4,4)$	$[786.97, -293.42]$	-102.70	1.0	t5,t5,t4,t4,t4,t4,t4	t4,t5 {Mx}
5	$(8,0,11,10,0,5,0)$	$[-1145.64, 199.42]$	-19.32	1.0	fa,0,fa,fa,0,t5,0	{FA}
6	$(0,4,0,5,5,5,0)$	$[390.03, -301.55$	-14.79	0.0	0,t4,0,t5,t5,t5,0	—
7	$(2,2,2,2,2,2,2)$	$[-380.84, -289.07]$	-113.88	0.0	t2,t2,t3,t2,t2,t2,t2	—
8	$(3,3,1,3,3,3,3)$	$[-391.73, -322.98]$	-113.26	0.0	t3,t3,t2,t3,t3,t3,t3	—
9	$(4,5,3,4,4,4,0)$	$[793.25, -305.67]$	-42.75	0.0	t5,t5,t4,t4,t4,t4,0	—
10	$(3,3,1,3,3,2,0)$	$[-390.84, -320.68]$	-49.16	0.0	t3,t3,t2,t3,t3,t2,0	—
11	$(1,1,0,1,1,1,0)$	$[-8.76, 319.23]$	-42.47	0.0	t1,t1,0,t1,t1,t1,0	—

TABLE VI
Composite Measurements, Corresponding True Measurement Probabilities, True Origin at Time $t = 6$

Meas. No.	S-tuple $z_i \in \Phi(t)$	ML Pos Est $\hat{\mathbf{x}}_p$	Neg Log-Like c_{z_i}	Probability P_z	LOS Components Target ID–FA	Composite Meas. Origin
1	$(1,1,0,1,1,1,1)$	$[194.10, 268.85]$	-103.08	0.0	t1,t1,0,t1,t1,t1,t1	—
2	$(2,5,1,2,2,2,3)$	$[-382.62, -484.96]$	-121.34	1.0	t2,t5,t2,t2,t2,t2,t3	t2
3	$(3,4,2,3,3,5,2)$	$[-501.81, -494.63]$	-112.65	1.0	t3,t4,t3,t3,t3,t5,t2	t3
4	$(4,2,3,5,5,0,0)$	$[298.47, -544.94]$	-37.80	0.0	t5,t2,t4,t5,t5,0,0	—
5	$(0,3,0,4,4,0,4)$	$[883.01, -518.17]$	-37.56	0.0	0,t3,0,t4,t4,0,t4	—
6	$(0,0,0,4,4,4,4)$	$[891.99, -498.80]$	-62.04	0.0	0,0,0,t4,t4,t4,t4	—
7	$(2,5,1,2,2,2,2)$	$[-382.68, -484.03]$	-118.44	0.0	t2,t5,t2,t2,t2,t2,t2	—
8	$(3,4,2,3,3,5,3)$	$[-501.39, -495.30]$	-114.84	0.0	t3,t4,t3,t3,t3,t5,t3	—
9	$(4,3,3,5,5,0,0)$	$[300.82, -546.00]$	-31.96	0.0	t5,t3,t4,t5,t5,0,0	—
10	$(3,3,2,3,3,5,2)$	$[-500.45, -498.76]$	-114.49	0.0	t3,t3,t3,t3,t3,t5,t2	—
11	$(4,2,3,5,0,0,0)$	$[273.72, -565.30]$	-30.47	0.0	t5,t2,t4,t5,0,0,0	—
12	$(1,1,0,1,1,1,0)$	$[191.83, 269.83]$	-42.45	0.0	t1,t1,0,t1,t1,t1,0	—
13	$(1,1,0,1,1,0,1)$	$[181.96, 264.54]$	-85.21	1.0	t1,t1,0,t1,t1,0,t1	t1
14	$(4,3,0,5,5,0,0)$	$[288.23, -515.21]$	-13.12	1.0	t5,t3,0,t5,t5,0,0	t5
15	$(0,2,3,4,4,0,4)$	$[868.71, -530.25]$	-82.61	1.0	0,t2,t4,t4,t4,0,t4	t4

TABLE VII
Composite Measurements, Corresponding True Measurement Probabilities, True Origin at Time $t = 7$

Meas. No.	S-tuple $z_i \in \Phi(t)$	ML Pos Est $\hat{\mathbf{x}}_p$	Neg Log-Like c_{z_i}	Probability P_z	LOS Components Target ID–FA	Composite Meas. Origin
1	$(1,1,0,1,1,1,1)$	$[394.48, 216.39]$	-102.07	0.999923	t1,t1,0,t1,t1,t1,t1	t1
2	$(2,2,1,2,2,3,2)$	$[-391.43, -696.42]$	-104.35	1.0	t2,t2,t2,t2,t2,t3,t2	t2
3	$(3,3,2,3,3,2,3)$	$[-595.24, -707.13]$	-111.55	1.0	t3,t3,t3,t3,t3,t2,t3	t3
4	$(4,5,3,5,5,5,0)$	$[190.08, -714.65]$	-51.27	1.0	t5,t5,t4,t5,t5,t5,0	t5
5	$(4,5,3,5,5,0,0)$	$[190.43, -716.78]$	-42.31	0.0	t5,t5,t4,t5,t5,0,0	—
6	$(2,2,1,2,2,2,2)$	$[-391.27, -696.46]$	-109.01	0.0	t2,t2,t2,t2,t2,t2,t2	—
7	$(3,3,2,3,3,3,3)$	$[-595.17, -706.66]$	-106.52	0.0	t3,t3,t3,t3,t3,t3,t3	—
8	$(1,1,0,1,1,1,0)$	$[392.55, 221.45]$	-42.13	0.0	t1,t1,0,t1,t1,t1,0	—
9	$(1,1,0,1,1,0,1)$	$[386.54, 221.94]$	-83.93	0.000077	t1,t1,0,t1,t1,0,t1	—

sets of composite measurement lists based on the S sets of LOS measurements. As a result, the dynamic 2-D phase could successfully associate the resulting composite measurements into tracks and discard an occasional false composite measurement.

The target motion models used in the state estimator (KF) were constant velocity and the measurement noise covariance matrices for the composite measurements were based on the derivations in Appendix A and had standard deviations 25–50 km. In Figs. 5 and 6 we provide rms plots for both position and velocity errors across 100 Monte Carlo simulations and averaged over the 5 targets. Clearly, as can be seen, fairly small and stable errors in position and velocity arise. Again, as before, because much of the work and cleaning up of

Composite Measurements, Corresponding True Measurement Probabilities, True Origin at Time $t = 8$

Meas. No.	S-tuple $z_i \in \Phi(t)$	ML Pos Est $\hat{\mathbf{x}}_p$	Neg Log-Like c_{z_i}	Probability P_z	LOS Components Target ID–FA	Composite Meas. Origin
1	$(1,1,0,1,1,1,1)$	$[597.48, 162.74]$	-94.27	1.0	t1,t1,0,t1,t1,t1,t1	t1
2	$(2,2,1,2,2,2,2)$	$[-394.87, -904.15]$	-104.42	1.0	t2,t2,t2,t2,t2,t2,t2	t2
3	$(3,3,0,0,3,3,3)$	$[-693.50, -912.80]$	-77.43	1.0	t3,t3,0,0,t3,t3,t3	t3
4	$(4,5,3,0,0,5,0)$	$[83.45, -869.44]$	-34.36	1.0	t5,t5,t4,0,0,t5,0	t5
5	$(0,0,0,4,4,4,4)$	$[1104.17, -906.94]$	-65.24	0.00001	0,0,0,t4,t4,t4,t4	—
6	$(2,2,13,2,0,3,2)$	$[-469.68, -865.55]$	-83.68	0.0	t2,t2,fa,t2,0,t3,t2	—
7	$(3,3,0,0,3,2,3)$	$[-686.90, -922.28]$	-85.81	0.0	t3,t3,0,0,t3,t2,t3	—

TABLE IX
Composite Measurements, Corresponding True Measurement Probabilities, True Origin at Time $t = 9$

Meas. No.	S-tuple $z_i \in \Phi(t)$	ML Pos Est $\hat{\mathbf{x}}_p$	Neg Log-Like c_{z_i}	Probability P_z	LOS Components Target ID–FA	Composite Meas. Origin
1	$(1,1,0,1,1,1,1)$	$[767.11, 127.91]$	-83.09	0.098367	t1,t1,0,t1,t1,t1,t1	t1
2	$(3,3,0,0,0,2,3)$	$[-788.48, -1171.09]$	-67.64	0.098366	t3,t3,0,0,0,t2,t3	t3
3	$(4,5,3,0,0,3,2)$	$[-247.31, -888.26]$	-77.66	0.098366	t5,t5,t4,0,0,t3,t2	t5
4	$(0,4,0,4,4,0,4)$	$[1118.65, -1093.11]$	-57.61	1.0	0,t4,0,t4,t4,0,t4	t4
5	$(0,2,1,2,2,2,2)$	$[-415.39, -1089.63]$	-95.56	0.0	0,t2,t2,t2,t2,t2,t2	—
6	$(0,0,2,3,3,3,3)$	$[-789.50, -1096.37]$	-75.28	0.0	0,0,t3,t3,t3,t3,t3	—
7	$(0,0,0,4,4,4,4)$	$[1197.61, -1108.90]$	-65.69	0.0	0,0,0,t4,t4,t4,t4	—
8	$(4,5,3,0,3,3,3)$	$[-236.08, -900.34]$	-62.56	0.0	t5,t5,t4,0,t3,t3,t3	—
9	$(0,0,1,2,2,2,2)$	$[-414.17, -1087.54]$	-82.07	0.901633	0,0,t2,t2,t2,t2,t2	t2
10	$(4,5,3,0,3,3,2)$	$[-234.96, -902.07]$	-66.04	0.0	t5,t5,t4,0,t3,t3,t2	—
11	$(1,1,0,1,1,1,0)$	$[801.39, 129.74]$	-39.78	0.0	t1,t1,0,t1,t1,t1,0	—
12	$(4,5,3,0,0,3,3)$	$[-253.43, -886.18]$	-75.69	0.901633	t5,t5,t4,0,0,t3,t3	t5,t3 {Mx}
13	$(1,1,0,1,1,0,0)$	$[796.84, 137.63]$	-33.86	0.901633	t1,t1,0,t1,t1,0,0	t1

TABLE X
Composite Measurements, Corresponding True Measurement Probabilities, True Origin at Time $t = 10$

Meas. No.	S-tuple $z_i \in \Phi(t)$	ML Pos Est $\hat{\mathbf{x}}_p$	Neg Log-Like c_{z_i}	Probability P_z	LOS Components Target ID–FA	Composite Meas. Origin
1	$(1,1,0,1,1,1,0)$	$[1022.61, 106.15]$	-28.64	0.0	t1,t1,0,t1,t1,t1,0	—
2	$(2,2,0,2,2,2,2)$	$[-413.00, -1285.32]$	-92.10	1.0	t2,t2,0,t2,t2,t2,t2	t2
3	$(3,3,2,3,3,3,3)$	$[-890.97, -1296.24]$	-107.29	1.0	t3,t3,t3,t3,t3,t3,t3	t3
4	$(0,0,0,4,4,4,4)$	$[1290.08, -1307.79]$	-65.77	1.0	0,0,0,t4,t4,t4,t4	t4
5	$(1,1,0,1,1,0,0)$	$[995.53, 74.61]$	-31.59	1.0	t1,t1,0,t1,t1,0,0	t1

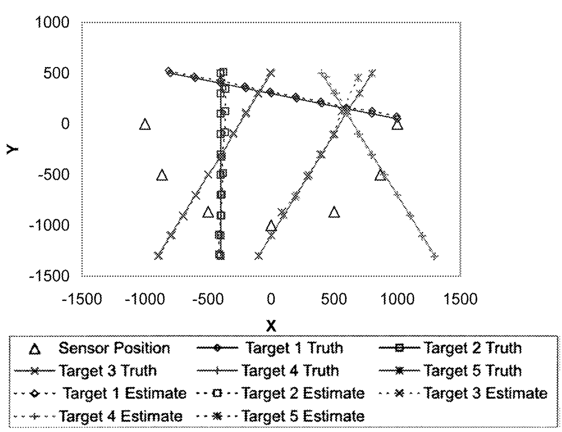

Fig. 4. Target tracks: true target positions, estimated positions from composite measurements processed.

the data was accomplished during the static m-best S-D phase, the dynamic 2-D phase of m-best S-D was fairly straightforward and non-stressing.

The 7 sensor scenario presents a formidable challenge to the S-D association algorithm, and pushes our m-best S-D algorithm to its limits. For the parallel performance, using a 4-processor SPARCstation 20, we obtained a speedup of 2.72 for an efficiency of 68%, where the standard definitions for speedup and efficiency are assumed, i.e.,

$$\text{speedup} = \frac{\tau_1}{\tau_p}, \qquad \text{efficiency} = \frac{\text{speedup}}{p} \tag{30}$$

where τ_1 denotes the sequential execution time utilizing 1 processor, and τ_p denotes the parallel execution time utilizing p processors.

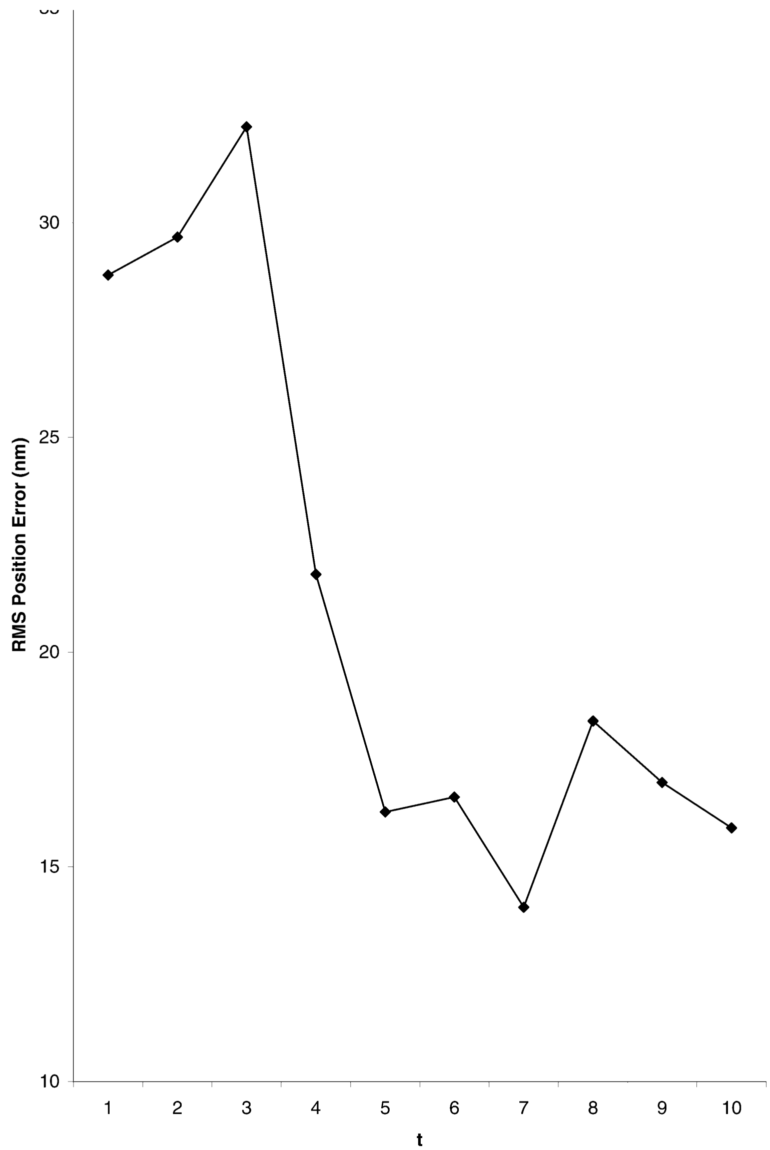

Fig. 5. RMS position plot across 100 Monte Carlo runs averaged over 5 targets.

VI. CONCLUSIONS

In this paper we described an efficient and novel approach to data association based on the m-best S-D assignment algorithm. We demonstrated the feasibility of the m-best S-D assignment algorithm for track formation and maintenance using a passive sensor multitarget tracking problem (operating in a type III configuration [5]). We showed how to efficiently (and nonenumeratively) extract sets of complete position "composite measurements" and determine their probabilities using a JPDA-like technique (based on the likelihoods associated with the feasible *joint events* corresponding to the (static) m-best S-D assignment solutions). We then used the series of composite measurement lists, along with their corresponding probabilities, in a dynamic 2-D assignment algorithm to estimate the states of the moving targets over time. We formulated the 2-D assignment cost coefficients using a likelihood function that incorporates the "true" composite measurement probabilities. Using a simulated passive sensor multitarget tracking problem, we showed that the m-best S-D assignment algorithm can perform well. Another significance of this work is that the m-best S-D assignment algorithm (in a sliding window mode) provides for an efficient implementation of a suboptimal MHT algorithm by obviating the need for a brute force enumeration of an exponential number of joint hypotheses.

APPENDIX A. COMPOSITE MEASUREMENT COVARIANCE DERIVATION

The solution of the dynamic 2-D assignment problem across lists of composite measurements (which is coupled with the target state estimator) requires the calculation of the measurement

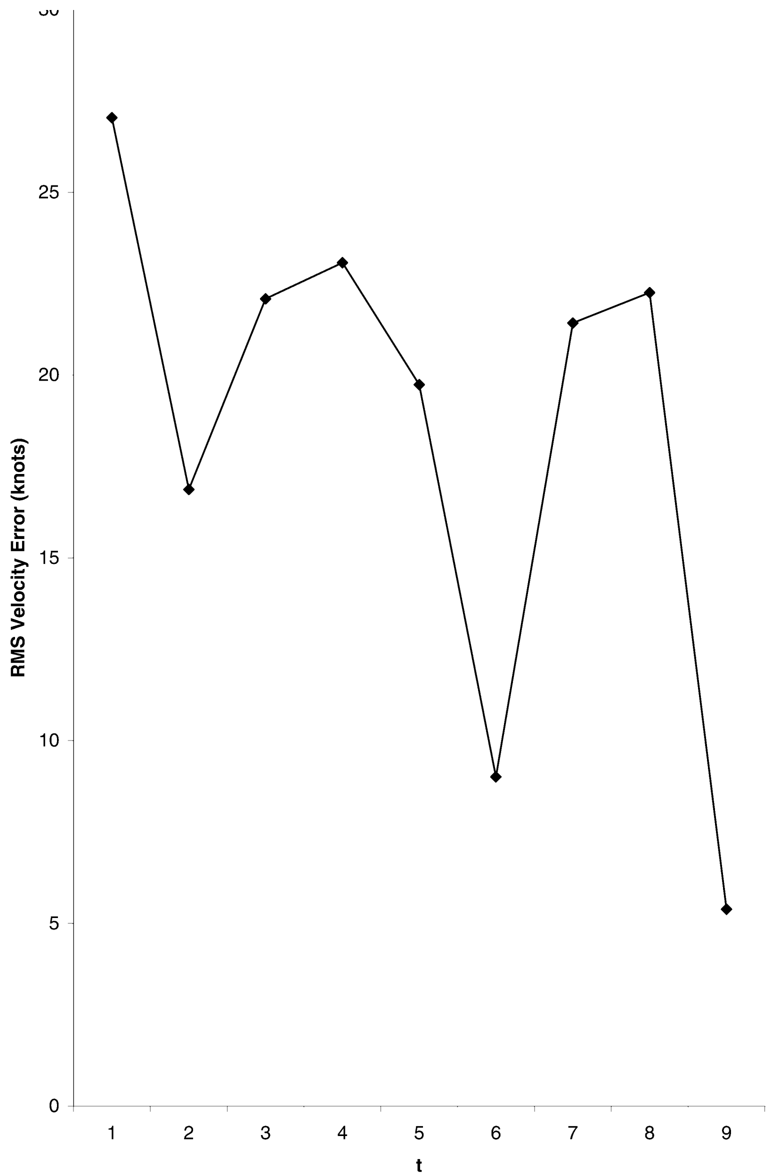

Fig. 6. RMS velocity plot across 100 Monte Carlo runs averaged over 5 targets.

covariance matrix $R(\cdot)$. In this work we use the Cramer–Rao lower bound on the covariance of the composite measurements [4] as the measurement noise covariance matrix entering into the dynamic state estimator. For a hypothesized localized target, having position estimate $\hat{\mathbf{x}}_p$ given by (5), the Fisher information matrix (FIM) is given by

$$J = r^{-1}\sum_{s=1}^{S}\mathbf{g}_{\hat{\mathbf{x}}_p}(s,\hat{\mathbf{x}}_p)\mathbf{g}_{\hat{\mathbf{x}}_p}(s,\hat{\mathbf{x}}_p)'$$

where S is the number of scans processed in the static S-D assignment problem, and

$$\mathbf{g}_{\hat{\mathbf{x}}_p}(s,\hat{\mathbf{x}}_p) \stackrel{\Delta}{=} \nabla_{\hat{\mathbf{x}}_p}\mathbf{g}(s,\hat{\mathbf{x}}_p).$$

For a target position estimate $\hat{\mathbf{x}}_p = [\eta_{\hat{x}_p}, \zeta_{\hat{x}_p}]'$, sensor position $\mathbf{y}_s = [\eta_{y_s}, \zeta_{y_s}]'$, and measurement

$$\theta = \tan^{-1}\left(\frac{\zeta_{\hat{x}_p} - \zeta_{y_s}}{\eta_{\hat{x}_p} - \eta_{y_s}}\right)$$

the expressions for the components of the gradient vector entering into the FIM are

$$g_\eta(s,\hat{\mathbf{x}}_p) = -\frac{\zeta_{\hat{x}_p} - \zeta_{y_s}}{(\eta_{\hat{x}_p} - \eta_{y_s})^2 + (\zeta_{\hat{x}_p} - \zeta_{y_s})^2}$$

$$g_\zeta(s,\hat{\mathbf{x}}_p) = \frac{\eta_{\hat{x}_p} - \eta_{y_s}}{(\eta_{\hat{x}_p} - \eta_{y_s})^2 + (\zeta_{\hat{x}_p} - \zeta_{y_s})^2}.$$

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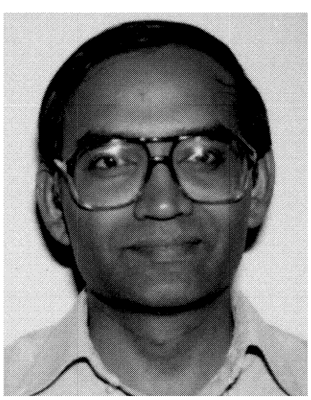
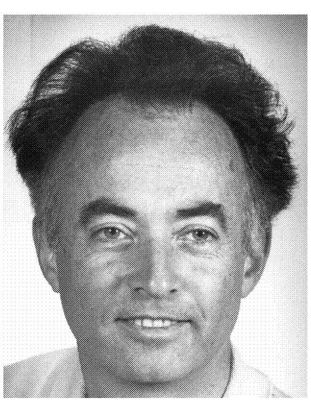
Dr. Popp has published over 60 technical papers and is the holder of two patents. He is a member of various honorary and technical societies, including Phi Beta Kappa and Upsilon Pi Epsilon, IEEE, and is listed in *Who's Who in the East* and *Who's Who in the World*. He is presently serving as Associate Editor for the *IEEE Transactions on Systems, Man, and Cybernetics* (SMC), and as Chairman of the IEEE SMC Society's Adaptive Computing Systems (ACS) technical committee. Dr. Popp's expertise is in multisensor-multitarget tracking, software engineering, parallel and distributed computing, adaptive computing systems, and computer networking.



Krishna R. Pattipati (S'77—M'80—SM'91—F'95) received the B.Tech degree in electrical engineering with highest honors from the Indian Institute of Technology, Kharagpur, in 1975, and the M.S. and Ph.D. degrees in systems engineering from the University of Connecticut, Storrs, in 1977 and 1980, respectively.

From 1980–1986 he was employed by ALPHATECH, Inc., Burlington, MA, where he supervised and performed research on fault-tolerant computer architecture modeling, human decision modeling, multi-target tracking, queuing networks, systems testing and diagnosis, and large-scale mixed-integer optimization. Since 1986, he has been with the University of Connecticut, where he is a Professor of Electrical and Systems Engineering.

Dr. Pattipati has published over 190 articles, primarily in the application of systems theory and optimization (continuous and discrete) techniques to large-scale systems. He has served as a consultant to ALPHATECH, Inc., IBM Research and Development, and Qualtech Systems, Inc. He was selected by the IEEE Systems, Man, and Cybernetics Society as the Outstanding Young Engineer of 1984, and received the Centennial Key to the Future award. He was elected a Fellow of the IEEE in 1995 for his contributions to discrete-optimization algorithms for large-scale systems and team decision making. He won the best technical paper awards at the 1985, 1990, and 1994 IEEE AUTOTEST Conferences, and at the 1997 Command and Control Conference. Dr. Pattipati is currently Editor of the *IEEE Transactions on SMC–Cybernetics* (Part B), and is the Vice-President for Technical Activities of the IEEE SMC Society.



Yaakov Bar-Shalom (S'63—M'66—SM'80—F'84) was born on May 11, 1941. He received the B.S. and M.S. degrees from the Technion, Israel Institute of Technology, in 1963 and 1967 and the Ph.D. degree from Princeton University, Princeton, NJ, in 1970, all in electrical engineering.

From 1970 to 1976 he was with Systems Control, Inc., Palo Alto, CA. Currently he is School of Engineering Distinguished Professor in the Dept. of Electrical and Systems Engineering and Director of the ESP Lab (Estimation and Signal Processing) at the University of Connecticut. His research interests are in estimation theory and stochastic adaptive control and he has published over 230 papers in these areas. In view of the causality principle between the given name of a peron (in this case, "(he) will track", in the modern version of the original language of the Bible) and the profession of this person, his interests have focused on tracking.

He coauthored the monograph *Tracking and Data Association* (Academic Press, 1988), the graduate text *Estimation and Tracking: Principles, Techniques and Software* (Artech House, 1993), the text *Multitarget-Multisensor Tracking: Principles and Techniques* (YBS Publishing, 1995), and edited the books *Multitarget-Multisensor Tracking: Applications and Advances* (Artech House, Vol. I 1990; Vol. II 1992). He has been elected Fellow of IEEE for "contributions to the theory of stochastic systems and of multitarget tracking." He has been consulting to numerous companies, and originated the series of Multitarget-Multisensor Tracking short courses offered via UCLA Extension, at Government Laboratories, private companies, and overseas. He has also developed the commercially available interactive software packages MULTIDAT TM for automatic track formation and tracking of maneuvering or splitting targets in clutter, PASSDAT TM for data association from multiple passive sensors, BEARDAT TM for target localization from bearing and frequency measurements in clutter, IMDAT TM for image segmentation and target centroid tracking and FUSEDAT TM for fusion of possibly heterogeneous multisensor data for tracking. During 1976 and 1977 he served as Associate Editor of the *IEEE Transactions on Automatic Control* and from 1978 to 1981 as Associate Editor of *Automatica*. He was Program Chairman of the 1982 American Control Conference, General Chairman of the 1985 ACC, and Co-Chairman of the 1989 IEEE International Conference on Control and Applications. During 1983–1987 he served as Chairman of the Conference Activities Board of the IEEE Control Systems Society and during 1987–1989 was a member of the Board of Governors of the IEEE CSS. Currently he is a member of the Board of Directors of the International Society of Information Fusion. In 1987 he received the IEEE CSS Distinguished Member Award. Since 1995 he is a Distinguished Lecturer of the IEEE AESS. He is co-recipient of the M. Barry Carlton Award for the best paper in the *IEEE Transactions on Aerospace and Electronic Systems* in 1995, and the 1998 University of Connecticut AAUP Excellence Award for Research.